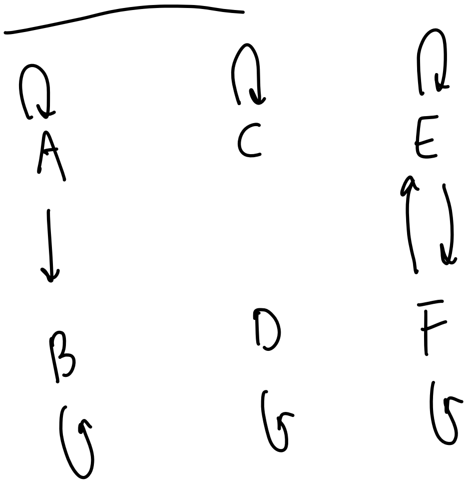
Examlet B Review Session

Problem 1



relation R defined by graph.

✓ reflexive
✗ irreflexive
✓ transitive → ERF, FRE → ERE ✓
FRE, ERF → FRF ✓
ARA, ARB → ARB ✓
✗ symmetric
✗ antisymmetric

Problem 2

Let R be a relation on $\mathbb{Z}$ such that xRy iff $x, y \in \mathbb{Z}$.

true for all set el.

Is R an equivalence relation?

reflexive? Yes → xRx true since $x \in \mathbb{Z}$.

transitive? $xRy \wedge yRz \to xRz$? yes always true

symmetric? xRy $(xFy) \to yRx$? yes always true.

what is [7]? if $x = 7$, what is y?

$7Ry$

$[7] = \mathbb{Z}, = [z \mid z \in \mathbb{Z}]$ $[\mathbb{Z}]$ → can we say this?

$[7] = [5] = [3] \ldots$

Problem 3

Given relation T on $\mathbb{N}$, aTb iff $\exists k \in \mathbb{N}$ s.t. $a = b + 2k$.

Prove T is antisymmetric & transitive.

antisymmetric: we want to show that if xRy and yRx, then $x=y$.

Let $x,y \in \mathbb{N}$ and let xTy and yTx.

By def'n of T, $\exists\, k \in \mathbb{N}$ such that $x = y + 2k$.

$\exists\, j \in \mathbb{N}$ such that $y = x + 2j$.

must use diff variables

By algebra, $x = x + 2j + 2k$

$-2j = 2k$

$-j = k$ $\quad$ $j,k \in \mathbb{N}$, so this is only true when $j = k = 0$.

Then, $x = y + 2(0) \longrightarrow x = y$. So T is antisymmetric.

transitive: we want to show that if xTy and yTz, then x z

Let $x,y,z \in \mathbb{N}$, and suppose xTy and yTz.

By def'n of T, $\exists\, k \in \mathbb{N}$ such that $x = y + 2k$

$\exists\, j \in \mathbb{N}$ such that $y = z + 2j$

By algebra, $x = z + 2j + 2k = x = z + 2(j+k)$

we know $j+k \in \mathbb{N}$, call $j+k = h$.

Then, $x = z + 2h$ where $h \in \mathbb{N}$, so

xTz.

Problem 4

which is true? false?

$\forall x \in \mathbb{Z}, \exists y \in \mathbb{N}, x^2 = y$ ✓

$\exists x \in \mathbb{Z}, \forall y \in \mathbb{N}, x^2 = y$ ✗

Problem 5

Let $\mathbb{N}_{>1}$ be natural numbers > 1.

$f: \mathbb{N}_{>1} \to \mathbb{N}_{>1}$ $\quad f(n) = \min\{x \in \mathbb{N}_{>1} \mid x \text{ divides } n\}$

$\mathbb{N} - \{0,1\}$ $\quad f(10) = \min\{x \in \mathbb{N}_{>1} \mid x \text{ divides } 10\}$

$f(10) = \min\{2,5,10\}$

$f(10) = 2$

a) is it a function? Yes, exactly one output for each input
↳ in co-domain ↳ in domain

b) domain: $\mathbb{N}_{>1}$
co-domain: $\mathbb{N}_{>1}$

c) what is the image of f? prime numbers not onto
since image $\neq$ co-domain

d) what is the pre-image of 3?
$\{3, 9, 15 \ldots\}$
odd multiples of 3

Problem 6

$f: \mathbb{Z}^+ \times \mathbb{Z}^+ \to \mathbb{Q}^2$ $\quad f(x,y) = \left(\frac{x}{y}, x+y\right)$

Prove f is one to one.

Want to show $f(a,b) = f(c,d) \to (a,b) = (c,d)$

Let $(a,b), (c,d) \in \mathbb{Z}^+ \times \mathbb{Z}^+$, and suppose $f(a,b) = f(c,d)$.

By def'n of f, $\left(\frac{a}{b}, a+b\right) = \left(\frac{c}{d}, c+d\right)$.

So, $\frac{a}{b} = \frac{c}{d}$, and $a+b = c+d$.

$a = \frac{bc}{d}$

$\frac{bc}{d} + b = c+d$

$bc + bd = cd + d^2$

$b(c+d) = d(c+d)$

since $c+d \neq 0$ $(c,d \in \mathbb{Z}^+)$, I can divide by $(c+d)$

so, $b = d$.

Then $a + d = c + d$

$a = c$.

Therefore, $(a,b) = (c,d)$. And f is one to one.

Problem 7

Suppose $f: \mathbb{Z} \to \mathbb{Z}$ is onto.

Define $g: \mathbb{Z}^2 \to \mathbb{Z}$ as $g(x,y) = f(x-7) \cdot f(y)$.

simplify this

Prove g is onto.

Let k be an arbitrary element of $\mathbb{Z}$. We will find a preimage.

Since f is onto, and 1 is an element of $\mathbb{Z}$, I know there exists a preimage in $\mathbb{Z}$. call this preimage a. So $f(a) = 1$.

$a \in \mathbb{Z}$, so $a+7$ is also $\in \mathbb{Z}$.

Then $g(a+7, y) = f(a+7-7) \cdot f(y) = f(a) \cdot f(y) = f(y)$

DO NOT SAY: g is equal to an onto function f, so g is onto.

Since f is onto, and $k \in \mathbb{Z}$, there exist a preimage to k, $b \in \mathbb{Z}$.

Such that $f(b) = k$.

So, $g(a+7, b) = f(b) = k$. $(a+7, b) \in \mathbb{Z}^2$, so g is onto.